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INVESTIGATION OF THE HEAT AND MASS TRANSFER DURING SUBLIMATION OF ICE BY THE METHOD OF "THERMAL SHOCK"

D. P. Lebedev and Lieh-Kue Ki

UDC 536.422.1

Experimental results are presented and computational dependences are obtained for the heat and mass transfer during sublimation of ice.

The experimental and theoretical investigations of the ice sublimation mechanism in a vacuum known at this time [1-5] for thermoradiation energy fluxes supplied to the sublimation zone examine the "stationary" sublimation process (first period) characterized by a constant sublimation rate and a constant temperature distribution in the specimen with time.

In reality, the process of sublimation parameter buildup (the heating period) [6] precedes the first period. This period is ordinarily eliminated completely [3-5, 13] in the traditional method of investigating the sublimation process. The complexity of these investigations is associated with the fact that the radiator itself is heated during the heating period and the radiation heat exchange of the subliming material with the radiator, with the walls of the vacuum chamber and other measuring attachments varies. In order to eliminate all secondary phenomena and to obtain more accurate experimental results, as well as to approximate the experimental results to the classical physical problem, we used a more perfect model with instantaneous insertion of an energy supply for the complex investigation of sublimation processes with thermoradiation supply of heat [6, 7]. This method of organizing the energy supply affords a possibility of obtaining well-founded experimental data,* which significantly expand the possibilities of a mathematical-physics analysis.

Experimental Model

The experimental model (Fig. 1) for the investigation of nonstationary sublimation processes in a vacuum was a structural modification of the model examined in [6], which consists of using two symmetric infrared radiators to organize the thermoradiation energy supply. The investigations were conducted at pressures from 1 to 10^{-3} mm Hg.

The experimental ice specimen 1 is cylindrical in shape (32 mm in diameter, 16 mm in thickness). To exclude uncheckable radiant fluxes from the specimen (vacuum chamber walls,

*We investigate in this paper the ice sublimation process under the effect of an "infinite" duration heat pulse which we call the "thermal shock" method.

Moscow Power Institute. All-Union Scientific-Research Biotechnical Institute, Moscow.
Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 29, No. 5, pp. 808-813, November, 1975. Original article submitted December 10, 1973.

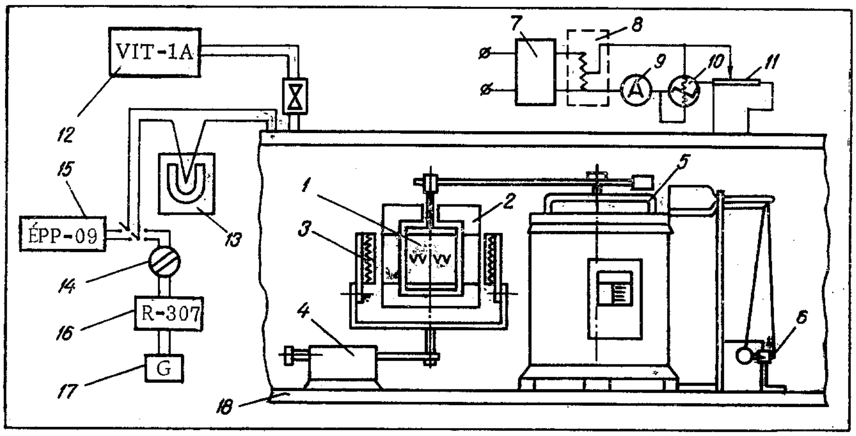

Fig. 1. Experimental model to investigate ice sublimation by the method of a two-sided "thermal shock": 1) ice specimen; 2) adiabatic jacket; 3) radiator; 4) electric motor; 5) scales; 6) orientation system; 7) stabilizer; 8) rheostat; 9) ammeter; 10) wattmeter; 11) rheostat; 12) vacuum meter; 13) Dewar vessel; 14) switch; 15) potentiometer; 16) potentiometer; 17) galvanometer; 18) vacuum chamber.

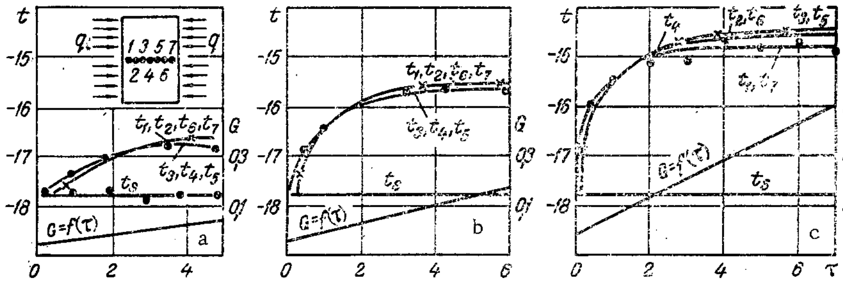

Fig. 2. Kinetics of the sublimation process in a vacuum during a two-sided "thermal shock": a) T_r = 50°C; b) T_r = 150°C; c) T_r = 250°C. Vacuum of 133 N/m^2; t, °C; G, g; τ, min.

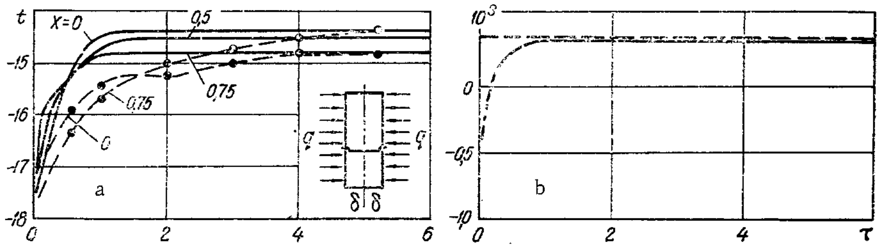

Fig. 3. Comparison between theoretical and experimental results: a) temperature change over the thickness of ice (solid curves, computation; dashed curves, experiment); b) time dependence of the sublimation intensity (solid curves, theory; dashed curves, experiment). $I \cdot 10^3$ $kg/m^2 \cdot sec$.

enclosures, and other adapters), a special adiabatic jacket 2, which was a volume filled with ice taking on an almost saturation temperature under vacuum conditions, was mounted around it.

The temperature of the experimental specimen was determined by Chromel—Copel thermocouples. The thermocouple emf was determined by an R-307 potentiometer with an M-125/3 galvanometer and ÉPP-09 recorder. The error in measuring the temperature by the recording potentiometer was not more than 0.5°C and not more than 0.2°C when using the R-307. A two-sided radiation energy supply ("thermal shock") was accomplished by using the displacement system of a "dark" radiator 3. In order to establish a given (identical) temperature preliminarily, the radiators occupied a position from which they irradiated an additional "compensation" specimen of ice (this specimen is not shown in Fig. 1). Upon the establishment of the given (identical) temperature the radiators were rotated to the ice specimen under investigation. The time to displace the radiators was 1-2 sec. The decrease in the mass of the subliming specimen was determined on a VLTK-500 balance. The experiments were carried out in a 50-350°C temperature range of the dark radiators. The results of the experiment are represented in Figs. 2 and 3.

Obtaining Computational Dependences for the Ice Sublimation Intensity in a Vacuum

The fundamental equations and general formulation of the mathematical problems taking account of the physical circumstance in a vacuum near the sublimation surface are examined in [5-9]. For many reasons the solution of these problems is usually of definite difficulty and requires the introduction of new simplifications in both the physical model of the process and in the computation scheme. In this case such simplifications are obtained in analyzing the kinetics of the ice sublimation process. It has been shown experimentally [5, 10, 13] that the temperature field of the ice is transformed during its heating and is set above the saturation temperature in conformity with the surface temperature. This circumstance permits determination of the temperature field from the solution of the problem of ordinary heating of an ice slab of initial thickness 2δ which has "frozen" (unsubliming) boundaries and later taking it to determine the sublimation intensity by using the heat-balance equation on the ice—vacuum interface.

The mathematical formulation of the problem of heating an ice slab by a flux of infrared energy in dimensionless form is*

$$\frac{\partial\theta}{\partial \mathrm{Fo}} = \frac{\partial^2\theta}{\partial X^2} + \mathrm{Po}\,\{\exp[-k\delta(1-X)] + \exp[-k\delta(1+X)]\} \tag{1}$$

with the initial and boundary conditions

$$\theta(X,\ 0) = 0;\quad \theta(\pm 1,\ \mathrm{Fo}) = 1;\quad \frac{\partial\theta}{\partial X}(0,\ \mathrm{Fo}) = 0. \tag{2}$$

The magnitude of the dimensionless temperature is defined as

$$\theta = \frac{t - t_s}{t_{s'} - t_s}. \tag{3}$$

The quantity $t_{s'} - t_s = \Delta t_s$ (ice overheating temperature) depends experimentally on the wavelength and intensity of the infrared radiation flux. For two-sided thermoradiation heat supply from "dark" radiators, the quantity Δt_s can be represented in the form of the dependence [7]

$$\Delta t_s = 0.7 + 0.35\cdot 10^{-2}\left[\frac{T_r}{100}\right]^4 \tag{4}$$

for a radiator temperature $T_r \le 250$°C, pressure $P \ge 6.66$ N/m², and heat flux $q \le 5000$ W/m².

Let us perform a Laplace transformation on the differential equation (1) and the boundary conditions (2):

$$\frac{d^2\bar\theta(X,\ p)}{dX^2} - p\bar\theta(X,\ p) + \frac{1}{p}\,\mathrm{Po}\,\{\exp[-k\delta(1-X)] + \exp[-k\delta(1+X)]\}, \tag{5}$$

$$\bar\theta(\pm 1,\ p) = \frac{1}{p},\quad \bar\theta(X,\ 0) = 0,\quad \frac{\partial\bar\theta}{\partial X}(0,\ p) = 0, \tag{6}$$

where

$$\bar\theta(X,\ p) = \int_0^\infty \theta(X,\ \mathrm{Fo})\exp(-p,\ \mathrm{Fo})\,d\mathrm{Fo}.$$

The approximate solution is found by the Bubnov—Galerkin method [11] in the family of functions of the form

*An ice specimen with more than 30 mm diameter can be taken as an infinite slab when using an adiabatic jacket.

$$\bar{\theta}(X,\ p) = \frac{1}{p} + \sum_{k=1}^{n} \bar{a}_k(p)\,[1 - X^{2k}]. \tag{7}$$

The solution has the form

$$\theta(X,\ \text{Fo}) = 1 + \varphi_1(X) + \varphi_2(X)\exp(-2.4674\text{Fo}) + \varphi_3(X)\exp(-25.5326\text{Fo}), \tag{8}$$

where $\varphi_1(X)$, $\varphi_2(X)$, and $\varphi_3(X)$ are determined by the equations

$$\begin{aligned}\varphi_1(X) &= (1.4063M\text{Po} - 0.5469N\text{Po}) + (-4.6875M\text{Po} + 3.2812N\text{Po})X^2 + (3.2812M\text{Po} - 2.7344N\text{Po})X^4;\\ \varphi_2(X) &= (-0.9877M\text{Po} + 0.1786N\text{Po} - 1.2721) + (1.2057M\text{Po} - 0.2180N\text{Po} + \\ &+ 1.5529)X^2 + (-0.2080M\text{Po} + 0.0394N\text{Po} - 0.2808)X^4;\\ \varphi_3(X) &= (-0.4136M\text{Po} + 0.3683N\text{Po} + 0.3972) + (3.4818M\text{Po} - 3.0632N\text{Po} - \\ &- 3.3028)X^2 + (-3.0632M\text{Po} + 2.6949N\text{Po} + 2.9056)X^4.\end{aligned} \tag{9}$$

The dimensionless coefficients M and N are determined from the equations

$$\begin{aligned}M &= \left\{\frac{2}{(k\delta)^2}[1 + \exp(-2k\delta)] - \frac{2}{(k\delta)^3}(1 - \exp(-2k\delta))\right\};\\ N &= \left\{\frac{4}{(k\delta)^2}[1 + \exp(-2k\delta)] - \frac{12}{(k\delta)^3}[1 - \exp(-2k\delta)] + \frac{24}{(k\delta)^4}[1 + \exp(-2k\delta)] - \frac{24}{(k\delta)^5}[1 - \exp(-2k\delta)]\right\}.\end{aligned} \tag{10}$$

Equations (8), (9), and (10) describe the temperature distribution (and the temperature field) in an ice slab with "frozen" boundaries of thickness 2δ. The ice sublimation intensity in a vacuum at the obtained temperature distribution is determined as

$$I = \rho\frac{d\xi}{d\tau} = \frac{\lambda}{c\delta}\,\frac{d\Phi}{d(\text{Fo})} = \frac{\lambda\Delta t_s}{\delta\cdot r}\,\frac{\partial\theta}{\partial X}\Big|_{X=\Phi}, \tag{11}$$

where $\Phi = \xi/\delta$.

Using (4), (8), (9), and (10), the sublimation intensity for a given "dark" radiator temperature can be determined from (11). Presented in Fig. 2 is an example of computing the temperature field in an ice slab by means of (8) for a thickness $2\delta = 0.016$ m, the coefficient of ray attenuation K = 400 1/m, dark radiator temperature 250°C, radiation flux on the ice surface q = 1214.67 W/m^2, coefficient of thermal conductivity 2.44 W/m•deg and $\Delta t_s = 2.3$°C. For these values the criterion is Po = 5.53 and the numbers are M = 0.1348 and N = 0.1835. The equations for $\varphi_1(X)$, $\varphi_2(X)$ and $\varphi_3(X)$ are

$$\begin{aligned}\varphi_1(X) &= 0.4938 - 0.1630X^2 - 0.3292X^4;\\ \varphi_2(X) &= -1.8277 + 2.2311X^2 - 0.4034X^4;\\ \varphi_3(X) &= 0.4590 - 3.8162X^2 - 3.3573X^4.\end{aligned} \tag{12}$$

The temperature field computed by means of (8) and (12) is represented in Fig. 3. As is seen from Fig. 3a, the computed temperature change has the identical character and yields identical values of the steady-state temperatures during a longer time than the experimental (5 min instead of 2 min). The discrepancy obtained between the computation and experiment can be explained by the following reasons:

1) Neglect of the change in specimen boundaries during heating;

2) The time dependence of Δt_s is not taken into account in (1) and the boundary conditions (4). The sublimation intensity (Fig. 3) for the example under consideration was determined by means of the equation

$$I = -\frac{\lambda\Delta t_s}{\delta r}[-1.6428 + 2.8486\exp(-2.4674\,\text{Fo}) + 5.7968\exp(-25.5326\text{Fo}). \tag{13}$$

The quantity Δt_s was computed by means of (4). As is seen from Fig. 3a, the computation of the ice sublimation intensity agrees well with the experimental results 1 min after the introduction of the energy supply.

Thus the scheme proposed for computation of the ice sublimation process in a vacuum within the framework of the assumptions taken yields especially good convergence for the quasistationary period (first period) and for the thermoradiation fluxes of infrared energy with wavelengths greater than 5 μ ("dark" radiator temperature not more than 300°C).

A considerable discrepancy in the results obtained should be expected during heating of the ice by infrared energy fluxes with wavelengths less than 5 μ ("dark" radiator temperature greater than 300°C) where a "hump-like" change in the temperature over the ice thickness was observed at the initial instant of the "thermal shock" [6, 13]. This change is much clearer for the short-wave infrared spectrum range (2-3 μ wavelength). These nonstationary processes, accompanied by infrared energy absorption in a layer of finite thickness and by the development of volume sublimation which occurs with significant stresses of the ice deformation, require additional theoretical study.

NOTATION

t, temperature; t_s, saturation temperature of water vapor in a vacuum; t_s', overheating temperature of the ice relative to the saturation temperature; Δt_s, temperature jump; λ, c, ρ, thermal conductivity, specific heat, and density of the ice; 2δ, initial thickness of the ice slab in a vacuum; k, coefficient of ray attenuation; t_r, T_r, "dark" radiator temperature; τ, time; r, heat of sublimation; $\theta = (t - t_s)/(t_s' - t_s)$, dimensionless temperature; $X = x/\delta$, dimensionless coordinate; $Fo = a\tau/\delta^2$, Fourier criterion; $Po = kT_0\delta^2/\Delta t_s\lambda$, Pomerantsev criterion

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